AP Calculus AB
Summer Packet 2018-2019

Dear future AP Calculus AB student:

Congrats! You have signed up for AP Calculus AB for 2018-2019 school year at Eastlake High School. AP Calculus is the culmination of all of the mathematics you have ever learned! It's truly an amazing experience that, at times, even amazes us at the amount of prior knowledge that comes into play to solve a problem. If you love mathematics and all of its magic, you are in the right place.

A component of hard work is perseverance. We expect successful AP Calculus students to possess a level of independence surrounding their learning. Engaging in study habits that enhance the "learn how to learn" model such as daily reflections, care and detail in the completion of each and every homework problem, self-correction and a drive to determine the solution, reading the textbook, accessing online resources, working with a study group, and seeking teacher assistance are all behaviors we believe make a student rise to their best and empower them to be future ready. The content in this course is dedicated by CollegeBoard and supported by universities across the country. In order to provide the opportunity for all students to reach their full potential on the AP exam in early May, this course will move at a fast pace given the vast amount of content to be covered. We do not require students to take the AP exam and we encourage students to experience a challenging, college level course as a means of preparation for that first quarter or semester of the college of your choice. Therefore, we highly recommend you to take to heart the afore mentioned "learn how to learn" habits as we, as educators, desire you to be at your best after four years at Eastlake.

There are certain math skills that have been taught to you over the previous years that are necessary to be successful in calculus. If you do not have these skills, you will consistently struggle to correctly solve problems next year and reach a solution, even though you understand the calculus concepts. It is frustrating for students when they are doing calculus, but are tripped up by the algebra or trigonometry. This summer packet is intended to help you brush up on and possibly relearn these topics. **On the first day of class in the fall, you must bring in all of the step-by-step work for each problem in this packet being mindful of the organization by properly titling each section by topic and numbering each problem. The work will be collected an awarded 100 effort points based on the evidence provided along with the correct answer (use the keys provided to guide yourself). The understanding of these concepts will be assessed within a week from the beginning of the school year through two quizzes. These quizzes are feedback to you about your prerequisite knowledge. Depending on your results, you may need to engage in additional study as calculus is dependent on this prior knowledge.**

To make the most of this packet and to start the semester off right, we recommend you spend some quality time with the packet this summer. Do not try to finish it before school is out for the summer—we want the topics to be fresh in your minds in the fall! Do not attempt to do it all the night before the first day of classes—you will find it a daunting task! We also recommend that you do not rely on your calculator. Almost all problems should be possible to solve using paper and pencil.

Looking forward to see you in the fall,

Mrs. Simon and Mrs. Timofte

AP Calculus AB Summer Packet Name_____

INSTRUCTIONS: *Where applicable, put your solutions in interval notation. Do not use any calculator (except on Topic 15:#5). Please do all work on separate paper and do the problems in order.* ***SHOW ALL WORK!*** *We've given you the answers—we care about the process that gets you there.*

Topic 1: Fractional and Negative Exponents

Simplify, using positive only exponents.

1. $2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right]^{-3}$ 2. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ 3. $\frac{\frac{1}{2}(2x+5)^{\frac{3}{2}}}{\frac{3}{2}}$ 4. $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

* skip

Topic 2: Domain

Find the domain of the following functions.

1. $y=\frac{x^2-5x-6}{x^2-3x-18}$ 2. $y=\frac{\sqrt{2x-9}}{2x+9}$ 3. $y=\sqrt{x^2-5x-14}$ 4. $y=\log(2x-12)$

Topic 3: Solving Inequalities

Write the following absolute value function as a piece-wise function.

1. $y=|2x-4|$

Solve the following absolute value inequalities.

2. $|x-3|\le 4$ 3. $|3x-4|>-2$

Solve the following quadratic inequalities.

4. $x^2-3x\ge 10$ 5. $x^3+4x^2-x\ge 4$ 6. $2\sin^2 x\ge \sin x,\ 0\le x<2\pi$

* skip

Solve the following rational inequality.

7. $\frac{2x-1}{3x-2}\le 1$

Topic 4: Even and Odd Functions

Show algebra to determine if the relation is even, odd, or neither.

1. $f(x)=2x^2-7$ 2. $f(x)=-4x^3-2x$ 3. $f(x)=4x^2-4x+4$

Topic 5: Function Transformation

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of :

1. $f(x) - 4$
2. $f(x - 4)$
3. $-f(x + 2)$
4. $5f(x) + 3$
5. $f(2x)$
6. $|f(x)|$

Topic 6: Factor theorem

*Using synthetic division, factor as indicated. *Know long division skill as well as it does appear in our AP Calculus work.*

(a) $x^3 - 4x^2 + 2x + 1 = (x - 1)(\quad)$

(b) $2x^3 + 5x + 7 = (x + 1)(\quad)$

(c) $x^4 - 3x^3 + x^2 + x + 2 = (x - 2)(\quad)$

(d) $4x^4 + 3x^2 - 1 = (2x - 1)(\quad)$

Topic 7: Special Factorization

Factor completely.

1. $27x^3 - 125y^3$
2. $x^4 + 11x^2 - 80$
3. $2x^2 + 50y^2 - 20xy$
4. $x^2 + 12x + 36 - 9y^2$
5. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$
6. $(3x+4)^{-3}(2x-5)^3 + (3x+4)^{-2}(2x-5)^2$
7. $\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2}$

Topic 8: Solving by Factoring or Quadratic Formula

Solve each equation.

1. $x^2 + 6x + 4 = 0$
2. $2x^2 - (x+2)(x-3) = 12$
3. $x - 10\sqrt{x} + 9 = 0$
4. $\frac{1}{x^2} - \frac{1}{x} = 6$
5. $x^3 + 2x^2 - 3x - 6 = 0$

Topic 9: Asymptotes

For each function, find the equations of both the vertical and horizontal asymptote(s), if they exist.

1. $y = \frac{x+4}{x^2+1}$
2. $y = \frac{x^2-9}{x^3+3x^2-18x}$
3. $y = \frac{2x^3}{x^3-1}$

Graph each rational function by hand. Use x,y intercepts to guide yourself.

Recall how to find a removable discontinuity (hole) in the graph. Confirm that each of these 3 rational function graphs <u>do or do not</u> have a hole. If so, locate the (x,y) coordinates of the hole.

Topic 10: Simplifying Expressions

Simplify.

1. $\dfrac{(x-1)(x+3)-(x+1)^2}{x+1}$

2. $\dfrac{1}{x+1}-\dfrac{1}{x-1}-\dfrac{2}{x^2-1}$

3. $\dfrac{x(-2x)}{2\sqrt{1-x^2}}+\sqrt{1-x^2}+\dfrac{1}{\sqrt{1-x^2}}$

4. $\dfrac{x^{-3}-x}{x^{-2}-1}$

5. $\dfrac{x^2-5x+6}{x^2-4x+4}$

6. $\dfrac{\sqrt{x^2+1}-\dfrac{1}{\sqrt{x^2+1}}}{x^2+1}$

Factor out the greatest common factor or as indicated.

7. $x^{-1}-2+x=x^{-1}(\quad)$

8. $\frac{x}{2}-6x^2=\frac{x}{2}(\quad)$

9. $\frac{1}{2x^2+4x}=\frac{1}{2x}(\quad)$

10. $e^{-x}-xe^{-x}+2x^2e^{-x}$

11. $2\sqrt{x}+6x^{3/2}$

12. $\sin x+\tan x=\sin x(\quad)$

Topic 11: Composition of Functions

If $f(x)=x^2$, $g(x)=2x-1$, and $h(x)=e^x$, find the following:

1. $f(g(2))$

2. $h(f(-1))$

3. $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

4. $g(f(x))$

5. $g(g(x))$

6. $f(h(x))$

Topic 12: Rationalizing Denominators and Numerators

Remove the sum or difference from the denominator by multiplying the numerator and denominator by the conjugate of the denominator.

1. $\dfrac{1}{1-\cos x}$

2. $\dfrac{x}{1-\sqrt{x^2+1}}$

3. $\dfrac{2}{x+\sqrt{x^2+1}}$

Topic 13: Logarithms and Exponentials

Simplify.

1. $\log_2 5+\log_2(x^2-1)-\log_2(x-1)$

2. $2\log_4 9-\log_2 3$

3. $3^{2\log_3 5}$

Expand using log properties. Simplify as needed.

4. $\log_5\left(\dfrac{\sqrt{x}y^3}{5z^4}\right)$

5. $\ln\left(\dfrac{7e}{(e-3)^2}\right)$

6. $\ln\left(\dfrac{e^4\sqrt{(e^2+6)}}{(x+2)^5}\right)$

Topic 14: Unit Circle

Evaluate each trigonometric function. No calculator.

1. $\sin \pi$
2. $\csc \frac{3\pi}{4}$
3. $\tan \frac{5\pi}{6}$
4. $\cot \frac{-4\pi}{3}$
5. $\sec 0$
6. $\cos \frac{-7\pi}{6}$

Evaluate each inverse trigonometric function. No calculator.

7. $\arcsin -\frac{\sqrt{3}}{2}$
8. $\operatorname{arcsec} \frac{2\sqrt{3}}{3}$
9. $cot^{-1}\ 0$

Evaluate each trigonometric function. No calculator.

10. $\tan(\operatorname{arcsec} -1)$
11. $\sec(\arcsin \frac{1}{2})$
12. $\operatorname{arccot}(\cos \pi)$

Topic 15: Solving Trigonometric Equations

Solve each equation on the interval [0, 2π). Please use a calculator to complete #5.

1. $\cos^2 x = \cos x$
2. $4 \sin^2 x = 1$
3. $2 \sin^2 x + \sin x = 1$
4. $2 \sin x \cos x + \sin x = 0$
5. $8 \cos^2 x - 2 \cos x = 1$
6. $\sin^2 x - \cos^2 x = 0$

Topic 16: Trigonometric Functions and Their Graphs

- *Be able to sketch each of the 6 trig functions, including domain, range, asymptotes, and period.*
- *Be able to determine the restricted domain of a trig function such that it's inverse will also be a function.*
- *Know how to sketch and discuss a transformed trig function with amplitude, period, and horiztonal and vertical shifts.*

Skip

Topic 17: Solving for Indicated Variable

Solve each equation for the indicated variable.

1. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, for a
2. $A = 2\pi r^2 + 2\pi rh$, for $r > 0$
3. $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$, for x

Skip

Topic 18: Equations of Lines

Determine the equation of each line:

1. the line through (-1, 3) and (2, -4)
2. the line through (-1, 2) and perpendicular to the line $2x - 3y + 5 = 0$
3. the line through (2, 3) and the midpoint of the line segment from (-1, 4) to (3, 2)

Topic 19: Conic Sections

- *Know the parent graphs and the equations for a circle, half-circle, ellipse, hyperbola and parabola (x and y direction).*

Topic 20: The Difference Quotient

Simplify $\frac{f(x+h)-f(x)}{h}$, *where:*

1. $f(x)=2x+3$
2. $f(x)=\frac{1}{x+1}$
3. $f(x)=x^2$

Topic 21: Trigonometric Identities

You should know the following identities and be able to use them to manipulate a trigonometric expression into a more simplified form.

1. $\sin(-x)=-\sin x$
2. $\cos(-x)=\cos x$
3. $\sin^2 x+\cos^2 x=1$
4. $\sin 2x=2\sin x\cos x$
5. $\cos 2x=\cos^2 x-\sin^2 x$
6. $\cos 2x=2\cos^2 x-1$
7. $\cos 2x=1-2\sin^2 x$

Pythagorean, Quotient & Reciprocal Practice

Verify the identity.

9 $\cos^2 t-\sin^2 t=2\cos^2 t-1$

11 $\frac{\sin t}{\csc t}+\frac{\cos t}{\sec t}=1$

13 $(1+\sin\alpha)(1-\sin\alpha)=\frac{1}{\sec^2\alpha}$

21 $\csc\theta-\sin\theta=\cot\theta\cos\theta$

37 $\frac{1+\csc\beta}{\sec\beta}-\cot\beta=\cos\beta$

45 $\frac{\cos\beta}{1-\sin\beta}=\sec\beta+\tan\beta$ * skip

49 $\frac{\cot u-1}{\cot u+1}=\frac{1-\tan u}{1+\tan u}$ * skip

53 $\tan^4 k-\sec^4 k=1-2\sec^2 k$ * skip

65 $(a\cos t-b\sin t)^2+(a\sin t+b\cos t)^2=a^2+b^2$

83 $\log 10^{\tan t}=\tan t$

85 $\ln\cot x=-\ln\tan x$

Double Angle Practice

A. $\sin(2\theta)=\frac{2\tan\theta}{1+\tan^2\theta}$ * skip

B. $\sin 2t-\tan t=\tan t\cos 2t$ * skip

C. $\sin^2 x+\cos 2x=\cos^2 x$

Name _____

Algebraic Atrocities

by Susan Margulies
Revised by David Pleacher

Statement	True or False	Correction
1. $\frac{3}{a}+\frac{3}{b}=\frac{3}{a+b}$	_____	_____
2. $\frac{a+b}{c+d}=\frac{a}{c}+\frac{b}{d}$	_____	_____
3. $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$	_____	_____
4. $\frac{a}{b+c}=\frac{a}{b}+\frac{a}{c}$	_____	_____
5. $\frac{10t+u}{10u+v}=\frac{t}{v}$	_____	_____
6. $\frac{a}{b}=\frac{a^2}{b^2}$	_____	_____
7. $\frac{a+b}{b}=a$	_____	_____
8. $\frac{1}{a+b}+(a+b)^2=a+b$	_____	_____
9. $2a^{-1}=\frac{-1}{2a}$	_____	_____
10. $a^{-2}=-a^2$	_____	_____
11. $(a-b)^2=a^2-b^2$	_____	_____
12. $(a+b)^2=a^2+b^2$	_____	_____

13. $(a+b)^3 = a^3 + b^3$ _____ _____

14. $\sqrt{a^2} = a$ _____ _____

15. $\sqrt{a^2 + b^2} = a + b$ _____ _____

16. $\sqrt{a^2 - b^2} = a - b$ _____ _____

17. $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ _____ _____

18. $\frac{1}{3}(-6)^3 = -2^3$ _____ _____

19. $a^{\frac{2}{3}} = \frac{a^2}{a^3}$ _____ _____

20. $\frac{\sin a}{a} = \sin(1)$ _____ _____

21. $\frac{\sin 2a}{a} = \sin(2)$ _____ _____

22. $\sin(2A) = 2\sin(A)$ _____ _____

23. $\sin(A+B) = \sin(A) + \sin(B)$ _____ _____

24. $\cos(2A) = 2\cos(A)$ _____ _____

25. $\cos(A+B) = \cos(A) + \cos(B)$ _____ _____

26. $\log(a+b) = \log(a) + \log(b)$ _____ _____

27. If $a+b=0$, then either $a=0$ or $b=0$ _____ _____

28. If x(x-2)=24, then either $x=24$ or $x-2=24$ _____ _____

29. $a\,(bc) = (ab)\,(ac)$ _____ _____

30. If $\log(a) = b$, then $a = \frac{b}{\log}$ _____ _____

31. If $\sin(a) = b$, then $a = \dfrac{b}{\sin}$ _____ _____

32. If $\cos(a) = b$, then $a = \dfrac{b}{\cos}$ _____ _____

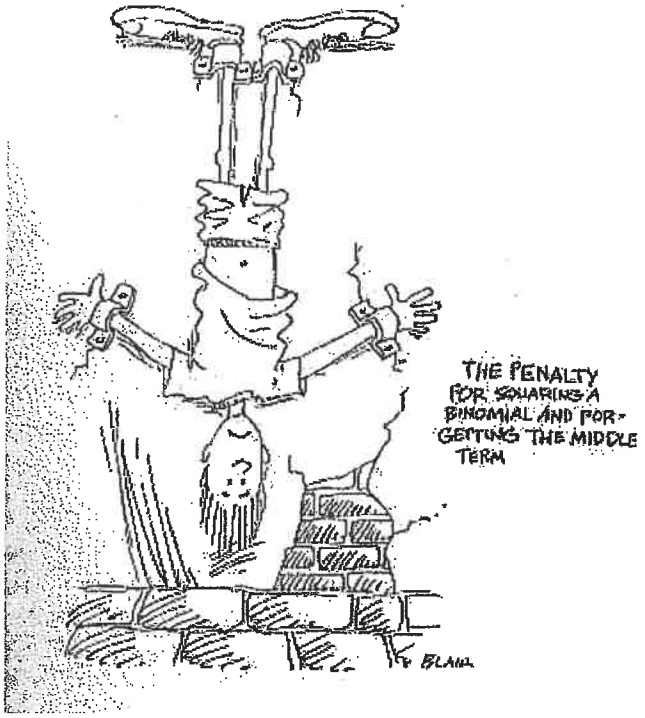
33. If $\tan(a) = b$, then $a = \dfrac{b}{\tan}$ _____ _____

34. $Sin^{-1}(x) = \dfrac{1}{\csc(x)}$ _____ _____

35. $Tan^{-1}(x) = \dfrac{1}{\cot(x)}$ _____ _____

36. $Cos^{-1}(x) = \dfrac{1}{\sec(x)}$ _____ _____

37. $Sin^{-1}(x) = \dfrac{1}{\sin(x)}$ _____ _____



Click here for answers

AP Calculus AB Summer Packet SOLUTIONS

Topic 1: 1) $\frac{-(2-x)^5}{2}$ 2) $\frac{2}{\sqrt[4]{(x-4)}}$ 3) $\frac{1}{3(2x+5)^{3/2}}$ 4) $\frac{1}{\sqrt{x^2+4xy+y^2}}$

Topic 2: 1) $(-\infty, -3) \cup (-3, 6) \cup (6, \infty)$ 2) $[9/2, \infty)$ 3) $(-\infty, -2] \cup [7, \infty)$ 4) $(6, \infty)$

Topic 3: 1) $y = \begin{cases} 2x-4, & x \geq 2 \\ 4-2x, & x < 2 \end{cases}$ 2) $[-1, 7]$ 3) $(-\infty, \infty)$ 4) $(-\infty, -2] \cup [5, \infty)$

5) $[-4, -1] \cup [1, \infty)$ 6) $[0] \cup [\pi/6, 5\pi/6] \cup [\pi, 2\pi)$ 7) $\left(-\infty, \frac{2}{3}\right) \cup [1, \infty)$

Topic 4: 1) even 2) odd 3) neither

Topic 5: 1) translated 4 units down 2) translated 4 units to the right

3) reflected over the x-axis and translated 2 units left

4) stretched vertically by a factor of 5 and translated up 3 units

5) stretched horizontally by a factor of ½

6) no change $(-\infty, -1) \cup (1, \infty)$, on $[-1, 1]$ the graph would be reflected over the x-axis

Topic 6: (a) $x^3 - 4x^2 + 2x + 1$

1	1	−4	2	1
		1	−3	−1
	1	−3	−1	0

$x^3 - 4x^2 + 2x + 1 = (x-1)(x^2 - 3x - 1)$

(b) $2x^3 + 5x + 7$

−1	2	0	5	7
		−2	2	−7
	2	−2	7	0

$2x^3 - 5x + 7 = (x+1)(2x^2 - 2x + 7)$

(c) $x^4 - 3x^3 + x^2 + x + 2$

2	1	−3	1	1	2
		2	−2	−2	−2
	1	−1	−1	−1	0

$x^4 - 3x^3 + x^2 + x + 2 = (x-2)(x^3 - x^2 - x - 1)$

(d) $4x^4 + 3x^2 - 1$

½	4	0	3	0	−1
		2	1	2	1
	4	2	4	2	0

$4x^4 + 3x^2 - 1 = \left(x - \frac{1}{2}\right)(4x^3 + 2x^2 + 4x + 2)$

$= (2x-1)(2x^3 + x^2 + 2x + 1)$

Topic 7: 1) $(3x-5y)(9x^2+15xy+25y^2)$ 2) $(x^2+16)(x^2-5)$ 3) $2(x-5y)^2$

4) $(x+6+3y)(x+6-3y)$ 5) $(x-3)^2(2x+1)^2(3x-2)$

6) $\frac{(2x-5)^2(5x-1)}{(3x+4)^3}$ 7) $\frac{(2x+1)^{3/2}(3x-1)}{15}$

Topic 8: 1) $x = -3 \pm \sqrt{5}$ 2) $x = -3$ or 2 3) $x = 1$ or 81 4) $x = -\frac{1}{2}$ or $\frac{1}{3}$

5) $x = -2, \sqrt{3}, -\sqrt{3}$

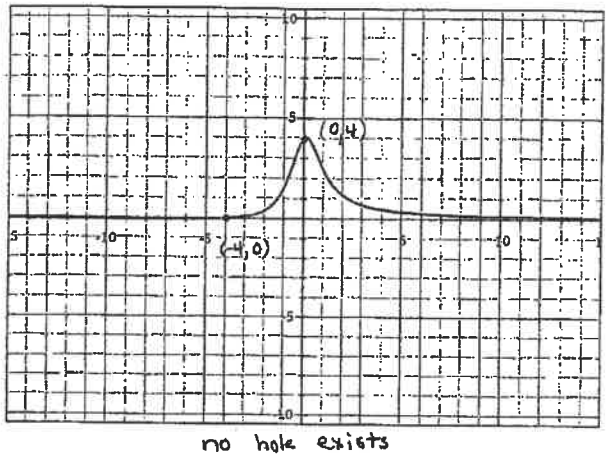
Topic 9:

1) VA: none; HA: $y = 0$

2) VA: $x = 0, x = -6$; HA: $y = 0$

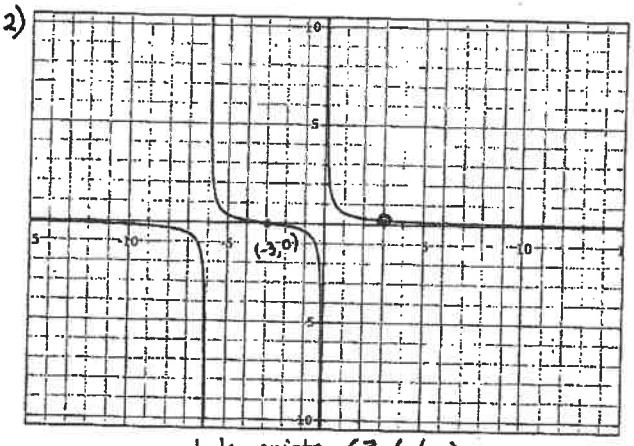
3) VA: $x = 1$; HA: $y = 2$

1)



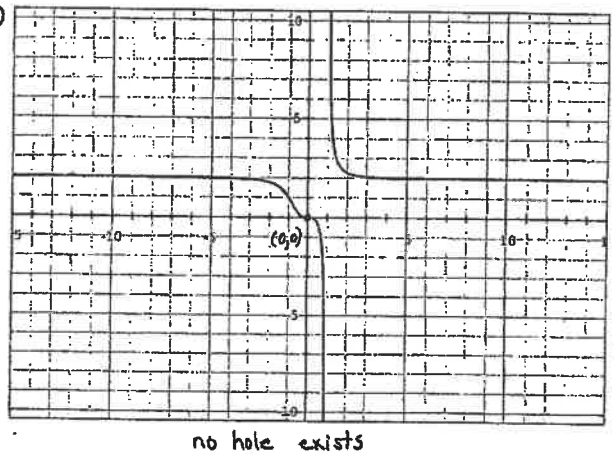
no hole exists

2)



hole exists $(3, 6/27)$

3)



no hole exists

Topic 10: 1) $\frac{-4}{x+1}$ 2. $\frac{(x-1)-(x+1)-2}{x^2-1}=\frac{-4}{x^2-1}$ 3. $2\sqrt{1-x^2}$

4) $\frac{1+x^2}{x}, x\neq -1, 1$ 5. $\frac{x-3}{x-2}$ 6. $\frac{x^2}{(x^2+1)^{3/2}}$

Topic 11: 1) 9 2) e 3) $2e-1$ 4) $2x^2-1$ 5) $4x-3$ 6) e^{2x}

Topic 12: 1) $\frac{1}{1-\cos x}=\left(\frac{1}{1-\cos x}\right)\left(\frac{1+\cos x}{1+\cos x}\right)$

$=\frac{1+\cos x}{1-\cos^2 x}=\frac{1+\cos x}{\sin^2 x}$

2) $\left(\frac{x}{1-\sqrt{x^2+1}}\right)\left(\frac{1+\sqrt{x^2+1}}{1+\sqrt{x^2+1}}\right)=\frac{x(1+\sqrt{x^2+1})}{1-(x^2+1)}$

$=\frac{x(1+\sqrt{x^2+1})}{-x^2}=\frac{1+\sqrt{x^2+1}}{-x}$

3) $\left(\frac{2}{x+\sqrt{x^2+1}}\right)\left(\frac{x-\sqrt{x^2+1}}{x-\sqrt{x^2+1}}\right)=\frac{2(x-\sqrt{x^2+1})}{x^2-(x^2+1)}=-2(x-\sqrt{x^2+1})$

Topic 13: 1) $\log_2 5(x+1), x>1$ 2) $\log_2 3$ 3) 25

4) $\frac{1}{2}log_5 x+3log_5 y-1-4log_5 z$ 5) $ln7+1-2ln(e-3)$

6) $4+\frac{1}{2}ln(e^2+6)-5ln(x+2)$

Topic 14: 1) 0 2) $\sqrt{2}$ 3) $\frac{-\sqrt{3}}{3}$ 4) $\frac{-\sqrt{3}}{3}$ 5) 1 6) $\frac{-\sqrt{3}}{2}$

7) $\frac{-\pi}{3}$ 8) $\frac{\pi}{6}$ 9) $\frac{\pi}{2}$ 10) 0 11) $\frac{2\sqrt{3}}{3}$ 12) $\frac{-\pi}{4}$

Topic 15: 1) $x=0,\frac{\pi}{2},\frac{3\pi}{2}$ 2) $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$ 3) $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{3\pi}{2}$

4) $x=0,\frac{2\pi}{3},\pi,\frac{4\pi}{3}$ 5) $x=\frac{\pi}{3},\frac{5\pi}{3}$, 1.823, 4.460 6) $x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

Topic 16: Trig Function Graphs: http://www.purplemath.com/modules/triggrph.htm

Transformed Trig Function Graphs: http://www.purplemath.com/modules/grphtrig.htm

Inverse Trig. Function Graphs:
http://www.vias.org/calculus/07_trigonometric_functions_03_03.html

Topic 17: 1) $a=\dfrac{bcx}{bc-cy-bz}$ 2) $r=\dfrac{\sqrt{h^2\pi^2+2\pi A}}{2\pi}-\dfrac{h}{2}$ 3) $x=\dfrac{\pi}{\pi-1}$

Topic 18: 1) $y=-\dfrac{7}{3}x+\dfrac{2}{3}$ 2) $y=-\dfrac{3}{2}x+\dfrac{1}{2}$ 3) $y=3$

Topic 19: Conic Sections and their Graphs:

http://www.mathplanet.com/education/algebra-2/conic-sections/equations-of-conic-sections

Topic 20: 1) 2 2) $\dfrac{-1}{(x+h+1)(x+1)}$ 3) $2x+h$

Topic 21:

CHAPTER 6

EXERCISES 6.1 PAGE 294

Exer. 1–87: Typical verifications are given for Exercises 1, 5, 9, . . . , 85.

1. $\cos\theta\sec\theta=\cos\theta\left(\dfrac{1}{\cos\theta}\right)=1$

5. $\dfrac{\csc x}{\sec x}=\dfrac{1/\sin x}{1/\cos x}=\dfrac{\cos x}{\sin x}=\cot x$

9. $\cos^2 t-\sin^2 t=\cos^2 t-(1-\cos^2 t)=2\cos^2 t-1$

13. $(1+\sin\alpha)(1-\sin\alpha)=1-\sin^2\alpha=\cos^2\alpha=1/\sec^2\alpha$

17. $\dfrac{\csc^2\theta}{1+\tan^2\theta}=\dfrac{\csc^2\theta}{\sec^2\theta}=\dfrac{1/\sin^2\theta}{1/\cos^2\theta}=\dfrac{\cos^2\theta}{\sin^2\theta}=\cot^2\theta$

21. $\csc\theta-\sin\theta=\dfrac{1}{\sin\theta}-\sin\theta=\dfrac{1-\sin^2\theta}{\sin\theta}=\dfrac{\cos^2\theta}{\sin\theta}$

$=\dfrac{\cos\theta}{\sin\theta}\cdot\cos\theta=\cot\theta\cos\theta$

37. $\dfrac{1+\csc\beta}{\sec\beta}-\cot\beta=\dfrac{1+\csc\beta-\cot\beta\sec\beta}{\sec\beta}$

$=\dfrac{1+\csc\beta-\dfrac{\cos\beta}{\sin\beta}\dfrac{1}{\cos\beta}}{\sec\beta}$

$=\dfrac{1+\csc\beta-\csc\beta}{\sec\beta}=\dfrac{1}{\sec\beta}=\cos\beta$

(45) $\dfrac{\cos\beta}{1-\sin\beta}\cdot\dfrac{1+\sin\beta}{1+\sin\beta}=\dfrac{\cos\beta(1+\sin\beta)}{1-\sin^2\beta}$

$=\dfrac{\cos\beta(1+\sin\beta)}{\cos^2\beta}=\dfrac{1+\sin\beta}{\cos\beta}$

$\therefore=\dfrac{1}{\cos\beta}+\dfrac{\sin\beta}{\cos\beta}=\sec\beta+\tan\beta$

(49) $\dfrac{\cot u-1}{\cot u+1}=\dfrac{(1/\tan u)-1}{(1/\tan u)+1}=\dfrac{(1-\tan u)/\tan u}{(1+\tan u)/\tan u}$

$=\dfrac{1-\tan u}{1+\tan u}$

(53) $\tan^4 k-\sec^4 k=(\tan^2 k-\sec^2 k)(\tan^2 k+\sec^2 k)$

$=(-1)(\sec^2 k-1+\sec^2 k)$

$=1-2\sec^2 k$

57 $(\sin^2\theta+\cos^2\theta)^3=(1)^3=1$

61 $\left(\dfrac{\sin^2 x}{\tan^4 x}\right)^3\left(\dfrac{\csc^3 x}{\cot^6 x}\right)^2=\left(\dfrac{\sin^6 x}{\tan^{12} x}\right)\left(\dfrac{\csc^6 x}{\cot^{12} x}\right)$

$=\dfrac{(\sin x\csc x)^6}{(\tan x\cot x)^{12}}=\dfrac{(1)^6}{(1)^{12}}=1$

(65) $(a\cos t-b\sin t)^2+(a\sin t+b\cos t)^2$

$=a^2\cos^2 t-2ab\cos t\sin t+b^2\sin^2 t$

$+a^2\sin^2 t+2ab\sin t\cos t+b^2\cos^2 t$

$=a^2(\cos^2 t+\sin^2 t)+b^2(\sin^2 t+\cos^2 t)$

$=a^2+b^2$

(85) $\ln(\cot x)=\ln(\tan x)^{-1}=-\ln(\tan x)$

Double Angle Work

A. $\sin 2\theta=\dfrac{2\tan\theta}{1+\tan^2\theta}$

$=\dfrac{\dfrac{2\sin\theta}{\cos\theta}}{\sec^2\theta}$

$=\dfrac{\dfrac{2\sin\theta}{\cos\theta}}{\dfrac{1}{\cos^2\theta}}$

$=\dfrac{2\sin\theta}{\cos\theta}\cdot\dfrac{\cos^2\theta}{1}$

$\therefore \sin 2\theta=2\sin\theta\cos\theta$

B. $\sin 2t-\tan t=\tan t\cos 2t$

$\dfrac{2\sin t\cos t}{1}-\dfrac{\sin t}{\cos t}=$

$\dfrac{2\sin t\cos^2 t-\sin t}{\cos t}=$

$\dfrac{\sin t(2\cos^2 t-1)}{\cos t}=$

$\dfrac{\sin t}{\cos t}\cdot\cos 2t=$

$\tan t\cdot\cos 2t=\tan t\cos 2t$

C. $\sin^2 x+\cos 2x=\cos^2 x$

$\sin^2 x+\cos^2 x-\sin^2 x=$

$\cos^2 x=\cos^2 x$

Name _____

Algebraic Atrocities

by Susan Margulies
Revised by David Pleacher

Statement	Correction
1. $\frac{3}{a}+\frac{3}{b}=\frac{3}{a+b}$	$\frac{3}{a}+\frac{3}{b}=\frac{3a+3b}{ab}$
2. $\frac{a+b}{c+d}=\frac{a}{c}+\frac{b}{d}$	$\frac{a+b}{c+d}=\frac{a}{c+d}+\frac{b}{c+d}$
3. $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$	TRUE
4. $\frac{a}{b+c}=\frac{a}{b}+\frac{a}{c}$	$\frac{a}{b+c}=\frac{a}{b+c}$ (cannot split up)
5. $\frac{10t+u}{10u+v}=\frac{t}{v}$	$\frac{10t+u}{10u+v}=\frac{10t+u}{10u+v}$ (cannot simplify)
6. $\frac{a}{b}=\frac{a^2}{b^2}$	$\frac{a}{b}=\frac{a}{b}$ (cannot square)
7. $\frac{a+b}{b}=a$	$\frac{a+b}{b}=\frac{a}{b}+1$ (cannot divide with +)
8. $\frac{1}{a+b}+(a+b)^2=a+b$	$\frac{1}{a+b}\times(a+b)^2=a+b$
9. $2a^{-1}=\frac{-1}{2a}$	$2a^{-1}=\frac{2}{a}$
10. $a^{-2}=-a^2$	$a^{-2}=\frac{1}{a^2}$
11. $(a-b)^2=a^2-b^2$	$(a-b)^2=a^2-2ab+b^2$
12. $(a+b)^2=a^2+b^2$	$(a+b)^2=a^2+2ab+b^2$

13. $(a+b)^3 = a^3 + b^3$ $\qquad$ $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

14. $\sqrt{a^2} = a$ $\qquad$ $\sqrt{a^2} = |a|$ ($\sqrt{\ }$ is only positive root)

15. $\sqrt{a^2 + b^2} = a + b$ $\qquad$ $\sqrt{a^2 + b^2} = \sqrt{a^2 + b^2}$ (cannot be simplified)

16. $\sqrt{a^2 - b^2} = a - b$ $\qquad$ $\sqrt{a^2 - b^2} = \sqrt{a^2 - b^2}$ (cannot be simplified)

17. $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ $\qquad$ $\sqrt{a+b} = \sqrt{a+b}$ (cannot be simplified)

18. $\frac{1}{3}(-6)^3 = -2^3$ $\qquad$ $\frac{1}{3}(-6)^3 = -72$

19. $a^{\frac{2}{3}} = \frac{a^2}{a^3}$ $\qquad$ $a^{\frac{2}{3}} = \sqrt[3]{a^2}$

20. $\frac{\sin a}{a} = \sin(1)$ $\qquad$ $\frac{\sin a}{a} = \frac{\sin a}{a}$ but $\lim_{a \to 0}\left(\frac{\sin a}{a}\right) = 1$

21. $\frac{\sin 2a}{a} = \sin(2)$ $\qquad$ $\frac{\sin 2a}{a} = \frac{\sin 2a}{a}$ or $\frac{2\sin a \cos a}{a}$

22. $\sin(2A) = 2\sin(A)$ $\qquad$ $\sin(2A) = 2\sin(A)\cos(A)$

23. $\sin(A+B) = \sin(A) + \sin(B)$ $\qquad$ $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

24. $\cos(2A) = 2\cos(A)$ $\qquad$ $\cos(2A) = \cos^2(A) - \sin^2(A)$

25. $\cos(A+B) = \cos(A) + \cos(B)$ $\qquad$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

26. $\log(a+b) = \log(a) + \log(b)$ $\qquad$ $\log(a \times b) = \log(a) + \log(b)$

27. If $a+b=0$, then either $a=0$ or $b=0$ $\qquad$ If $a+b=0$, then $a=-b$

28. If x(x-2)=24, then either $x=24$ or $x-2=24$ $\qquad$ NO! Only works if set = 0

29. $a\,(bc) = (ab)\,(ac)$ $\qquad$ $a\,(bc) = (ab)\,(c)$ (not distributive)

30. If $\log(a) = b$, then $a = \frac{b}{\log}$ $\qquad$ If $\log(a) = b$, then $a = 10^b$

31. If $\sin(a)=b$, then $a=\frac{b}{\sin}$ If $\sin(a)=b$, then $a=Sin^{-1}(b)$

32. If $\cos(a)=b$, then $a=\frac{b}{\cos}$ If $\cos(a)=b$, then $a=Cos^{-1}(b)$

33. If $\tan(a)=b$, then $a=\frac{b}{\tan}$ If $\tan(a)=b$, then $a=Tan^{-1}(b)$

34. $Sin^{-1}(x)=\frac{1}{\csc(x)}$ $Sin^{-1}(x)=Csc^{-1}\left(\frac{1}{x}\right)$ or $\sin(x)=\frac{1}{\csc(x)}$

35. $Tan^{-1}(x)=\frac{1}{\cot(x)}$ $Tan^{-1}(x)=Cot^{-1}\left(\frac{1}{x}\right)$ or $\tan(x)=\frac{1}{\cot(x)}$

36. $Cos^{-1}(x)=\frac{1}{\sec(x)}$ $Cos^{-1}(x)=Sec^{-1}\left(\frac{1}{x}\right)$ or $\cos(x)=\frac{1}{\sec(x)}$

37. $Sin^{-1}(x)=\frac{1}{\sin(x)}$ $Sin^{-1}(x)$ means Arcsin(x) but $(sin(x))^{-1}=\frac{1}{\sin(x)}$