<u>AP CALCULUS AB Summer Packet 2020-2021 Eastlake H.S.</u>

Dear Future AP Calculus AB Student,

Congrats! You have signed up for AP Calculus AB for the 2020-2021 school year at Eastlake High School. AP Calculus AB is the culmination of all the mathematics you have learned! It's truly an amazing experience that enhances *all* the mathematics you have learned through completely new concepts and applications emphasizing critical thinking, analysis, and reasoning coupled with advanced algebraic and trigonometry knowledge. If you love mathematics, get ready for a wild ride as this course will follow the College Board set curriculum. A quick pace will be followed to ensure completion of the required content and to ensure all students can have the opportunity to earn college credit with a passing score of a 3,4 or 5 on the College Board exam.

A big piece of being successful in this class is hard work and perseverance. We expect AP Calculus AB students to possess a level of independence surrounding their learning and engaging in study habits that *enhance* the "learn how to learn" model which include

- Engaging in daily reflections on your own learning
- Exhibiting care, attention, and detail in the completion of every homework problem
- Self-correction of mistakes, both in daily assignments and assessments, as mistakes will occur and are expected and the power of what can be learned from them isn't to be ignored
- Reading the textbook before, during, and after lesson presentations
- Accessing online resources to supplement your understanding
- Working in a study group
- Seeking teacher assistance during tutorials and Wolf Time

There are certain math skills that have been taught to you over the previous years that are necessary to be successful in *any* calculus course. If you do not have these skills, you will consistently struggle to correctly solve problems and reach solutions, even though you understand the calculus concept. The Summer Assignments enclosed are intended to help you brush up on and possibly relearn these essential skills. We have created a sampling of foundational skills in algebra and trigonometry that you can practice remotely over the summer, including an optional packet which can be found on the EHS school website under AP Calculus.

All the best!
Mrs. Simon (jsimon@lwsd.org) and Mrs. Timofte (mtimofte@lwsd.org)

What is in this packet?

- Student letter (you just read this)
- AP Calculus AB Foundational Pre-Requisite Skills overview
- Mandatory Summer Assignment Part 1 – Trigonometry: foundational skills required exercises to be done with paper & pencil to be submitted during the first week of classes. The answer key can be found at the end of the Mandatory part of the packet.
- Mandatory Summer Assignment Part 2 –Algebra: basic skills review to be completed on Khan Academy at 75% or above. Below you will see a table listing the title of each Khan Academy "practice" with a supporting video hyperlink for you to refresh your understanding if need be.
- Optional practice exercises and the step by step solutions to all these exercises.
- Expect 4-5 hours of work for the Mandatory Assignment (Part 1 and Part2) & 2-3 hours for the Optional Practice
- If you need any assistance with the summer assignments, please email the teachers at jsimon@lwsd.org or mtimofte@lwsd.org

Additional Teacher Support?

Please consider attending office hours via Microsoft Teams with an AP Calculus AB teacher on the following dates and times.

	Monday 8/17	***Tuesday 8/18***	***Wednesday 8/19***	***Thursday 8/20***
Time	1-2 pm	1-2 pm	1-2 pm	1-2 pm
Teacher	Mrs. Simon	Mrs. Timofte	Mrs. Simon	Mrs. Timofte
Teams Meeting link	Join Microsoft Teams Meeting	Join Microsoft Teams Meeting	Join Microsoft Teams Meeting	Join Microsoft Teams Meeting

What can you expect in the fall?

- Submit the Summer Assignment Part 1, neatly organized and with supporting work for each problem to your teacher.
- Teacher will confirm the completion of all assigned Khan Academy practices at 75% or above.
- Completion of the Summer Assignments Part 1 & Part 2 will be used to determine algebraic and trigonometric readiness and receive a grade in the grade book.

AP Calculus AB Foundational Pre-Requisite Skills

Algebra

- can manipulate with ease fractions, decimals and variables in a variety of settings including in equations and rational functions
- comfortable with all forms of factoring including quadratics, sum and difference of cubes, quartic and factoring by grouping
- completing the square
- synthetic and long division with polynomials
- add, subtract, multiply and divide radical expressions including rationalizing denominators
- use exponents and logarithms properties to simplify expressions
- solving equations involving logarithms and rational exponents

Graphing

- be familiar with the graphs of linear, absolute value, quadratic, cubic, quartic, logarithmic, exponential, and rational functions
- identify the domain and range of functions
- recognize end behaviors of graphs, vertical and horizontal asymptotes, and holes
- identify transformations and use them to graph

Trigonometry

- work with the basic six trig functions including manipulating them to simplify expressions and solve equations by finding all solutions
- know and apply a subset of trig identities
- Know the trig table of values - know the graphs of the six basic trig functions including their domain and range
- know how to find, without a calculator, the values of the inverse trig functions at chosen values

Mandatory Summer Assignment Part 1: Complete all required exercises on a separate sheet of paper with supporting work shown, use the answer key on the last page of this document to confirm your understanding and review the required trig formulas and facts given below. Be ready to submit your neatly organized and supporting work for each problem to your teacher during the first week of school for a completion grade in the grade book.

VI. Trigonometric Functions

Trig Facts: You will need to know these as you know your multiplication tables.
The values for sin, cos, and tan in Quadrant I should be *memorized cold*. The other Quadrants' values should not take you more than a few seconds to say when quizzed.
Every trig function takes an angle as input and returns a ratio as output.

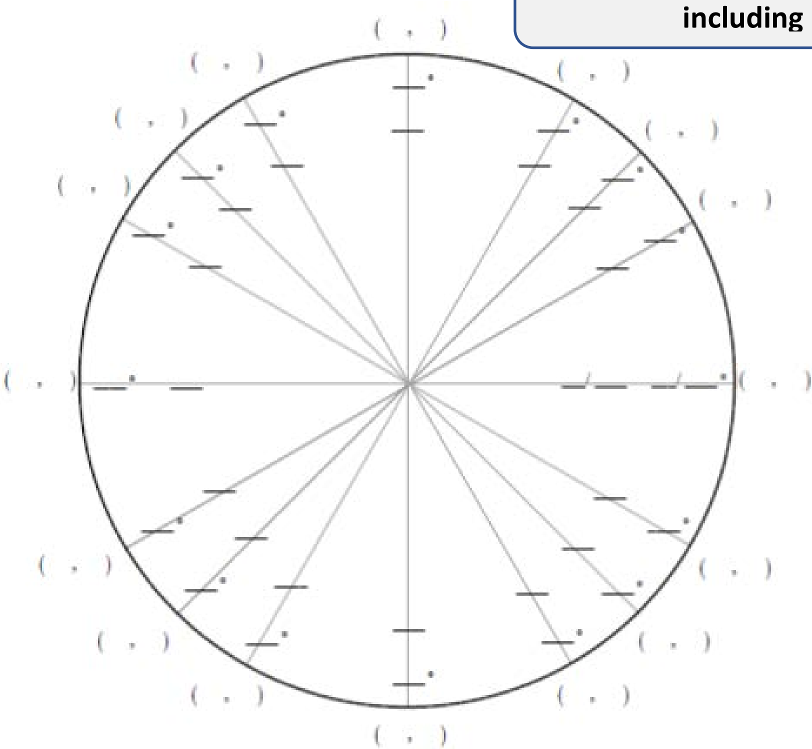
So every inverse trig function takes a ratio as input and returns ________ as output.

III. Fill out the following 16 point unit circle by finding the following:

1) The measures for each angle in radian and in degree.
2) The coordinate pair for each angle.

Know all points on the Unit Circle, including radians

Know trig values for Quadrant I



Fonction / Angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
SIN					
COS					
TAN					

- **Be able to sketch all 3 trig functions <u>and</u> their reciprocal functions, label their vertical asymptotes, and state their domains and ranges.**

- **Know the relationship between trig functions and their inverse functions and *why* their domains & ranges are switched and *why* the restricted domains of the trig functions are required to ensure the inverse trig function exists. Here is a summary of the inverse trig function's domains & ranges.**

Domain and Range of Trigonometry Functions

	Domain	Range
$\sin^{-1}$	[-1, 1]	[-π/2,π/2]
$\cos^{-1}$	[-1, 1]	[0,π]
$\tan^{-1}$	R	(-π/2,π/2)
cosec^{-1}	R - (-1, 1)	[-π/2,π/2] - {0}
$\sec^{-1}$	R - (-1, 1)	[0,π]-{π/2}
$\cot^{-1}$	R	(0,π)

Basic and Pythagorean Identities

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sin(x) = \frac{1}{\csc(x)}$$

$$\sec(x) = \frac{1}{\cos(x)} \qquad \cos(x) = \frac{1}{\sec(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)} \qquad \tan(x) = \frac{1}{\cot(x)} = \frac{\sin(x)}{\cos(x)}$$

$$\sin^2(t) + \cos^2(t) = 1$$

$$\tan^2(t) + 1 = \sec^2(t)$$

$$1 + \cot^2(t) = \csc^2(t)$$

Be comfortable using just these trig identities

$$\sin 2x = 2sinx\ cosx$$

$$\cos 2x = cos^2x - sin^2x = 2cos^2x - 1 = 1 - 2sin^2x$$

Mandatory Summer Assignment Part 1 Required Exercises (answers at the end of the mandatory part of the packet). Please complete on a separate sheet of paper in pencil and keep it neatly organized and provide supporting work for each problem as this work will be submitted to your teacher for a completion grade in the grade book during the first week of class.

A. Evaluate each trigonometric function for the value or angle given.

1. $\sec(2\pi)$
2. $\csc\pi$
3. $sin^{-1}(\frac{\sqrt{3}}{2})$
4. $cos^{-1}(\frac{-\sqrt{3}}{2})$
5. $\csc\frac{-7\pi}{6}$
6. $\sec\frac{-5\pi}{6}$
7. $\tan\frac{5\pi}{3}$
8. $\cot\frac{3\pi}{4}$
9. $\arccos\frac{-1}{2}$
10. $\arcsin\frac{-1}{2}$
11. $\arctan(\sqrt{3})$
12. $\arctan(\frac{\sqrt{3}}{3})$

Solving equations involving trigonometric functions:

Example: Solve the equation $2\,sin^2 x + 3sinx = -1$ on the interval $[0,2\pi]$

Consider sin x=u; the equation becomes: $\mathbf{2u^2 + 3u + 1 = 0}$ *or* $\mathbf{(2u+1)(u+1) = 0}$.
This equation has the solutions -1/2 and -1, so we need to solve on the interval $[0,2\pi]$ sin x= -1/2 and sin x = -1.
The solutions are : x= 7π/6, x= 11π/6, and x= 3π/2.

B. Solve the equation $2\,sin^2 x + sinx = 1$ on the interval $[0,2\pi]$
C. Solve the equation $2\,cos^2 x + cosx = 1$ on the interval $[0,2\pi]$
D. Solve the equation $\sin 2x = -1$ *on the interval* $[0,3\pi]$
E. Solve the equation 2sin 2x = 1 on the interval $[0,2\pi]$
F. Solve the equation $sin^2 x - sinx = 0$ on the interval $[0,2\pi]$
G. Solve the equation $cos^2 x - cosx = 0$ on the interval $[0,2\pi]$

H. Verify each identity.

1) $\frac{1+\csc x}{\sec x} - \cot x = \cos x$

2) $\frac{sec^2x-1}{sec^2x} = sin^2x$

3) $secx + tanx = \frac{cosx}{1-sinx}$

Mandatory Summer Assignment Part 2:

In order to receive the completion points for this Summer Assignment Part 2, you are required to complete the below listed "practices" on Khan Academy. Please do the following steps to access these assigned "practices":

Step #1: click on the unique class code for the AP Calculus Summer Assignment Part 2 to join the class titled **"EHS AP Calculus AB 2020-21"** https://www.khanacademy.org/join/EBT8YW34

Step #2: sign up as a member of Khan Academy, use your FULL NAME and SCHOOL EMAIL ADDRESS.

Step #3: locate the Assignments list for this class titled **"EHS AP Calculus AB 2020-21"** and begin your work.

Khan Academy Summer Packet Assignment Part 2

Title of Assignment "practice" on Khan Academy	Supporting Video Links on Khan Academy
Polynomials	
Factor using polynomial division	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-div/x2ec2f6f830c9fb89:poly-div-by-linear/v/factor-w-poly-div?modal=1
	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-div/x2ec2f6f830c9fb89:poly-div-by-linear/v/factor-w-poly-div-missing-term?modal=1

Factor higher degree polynomials	*Overview of Factoring of Polynomials:* https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-factor/x2ec2f6f830c9fb89:mono-factor/v/factor-high-deg-poly-intro?modal=1 *Higher Degree Polynomials:* https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-factor/x2ec2f6f830c9fb89:factor-high-deg/v/factor-high-deg-poly?modal=1 *Additional High Degree Polynomials* https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-factor/x2ec2f6f830c9fb89:factor-high-deg/v/factoring-perfect-square-polynomial?modal=1
Divide polynomials with remainders	https://www.khanacademy.org/math/algebra-home/alg-polynomials/alg-practice-dividing-polynomials-with-remainders/v/dividing-polynomials-with-remainders
Zeros of polynomials (with factoring)	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-graphs/x2ec2f6f830c9fb89:poly-zeros/v/polynomial-zeros-grouping?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-graphs/x2ec2f6f830c9fb89:poly-zeros/v/polynomial-zeros-common-factor?modal=1
Composite Functions	
Evaluate composite functions: graphs & tables	https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:composing/v/evaluating-composite-functions-using-tables?modal=1 AND https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:composing/v/evaluating-composite-functions-using-graphs?modal=1

Find composite functions	https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:composing/v/new-function-from-composition?modal=1 AND https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:composing/v/evaluating-composite-functions-example-1?modal=1
Inverse Functions	
Verify inverse functions	https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:verifying-inverse/v/verifying-function-inverses-by-composition?modal=1
Rational Exponents	
Fractional exponents	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:exp/x2ec2f6f830c9fb89:rational-exp/v/rewriting-roots-as-rational-exponents?modal=1
Logarithms	
Use the properties of logarithms	*Product Rule:* https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:logs/x2ec2f6f830c9fb89:log-prop/v/sum-of-logarithms-with-same-base?modal=1 *Power Rule:* https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:logs/x2ec2f6f830c9fb89:log-prop/v/logarithm-of-a-power?modal=1 *Multiple Steps using Properties of Logarithm:* https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:logs/x2ec2f6f830c9fb89:log-prop/v/using-multiple-logarithm-properties-to-simplify?modal=1

Transformations of Functions	
Identify function transformations	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:trans-all-together/v/shifting-and-reflecting-functions?modal=1
Even & Odd Functions	
Even and odd functions: Graphs and tables	*Graphs:* https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:symmetry/v/recognizing-features-of-functions-2-example-2?modal=1 *Tables:* https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:symmetry/v/even-and-odd-functions-tables?modal=1
Graphs of Functions	
Graphs of square and cube root functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:radical-graphs/v/graphing-square-and-cube-root?modal=1
Graphs of exponential functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:exp-graphs/v/transforming-exponential-graphs?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:exp-graphs/v/transforming-exponential-graphs-2?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:exp-graphs/v/graphing-exponential-functions-interactive?modal=1

Graphs of logarithmic functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:log-graphs/v/graphing-logarithmic-functions-1?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:log-graphs/v/graphing-logarithmic-functions-2?modal=1
Solving Equations	
Quadratic systems	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:eq/x2ec2f6f830c9fb89:quad-sys/v/line-and-parabola-system?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:eq/x2ec2f6f830c9fb89:quad-sys/v/systems-of-nonlinear-equations-3?modal=1
Rational Functions	
End behavior of rational functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-end-behavior/v/end-behavior-of-rational-functions?modal=1
Rational function points of discontinuity	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:discontinuities/v/discontinuities-of-rational-functions?modal=1
Analyze vertical asymptotes of rational functions	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:discontinuities/v/analyzing-vertical-asymptotes-of-rational-functions?modal=1
Add & subtract rational expressions: unlike denominators	https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-add-sub-intro/v/adding-rational-expression-w-unlike-denominators?modal=1 AND https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-add-sub-intro/v/subtracting-rational-expressions-w-unlike-denominators?modal=1

Piecewise Functions	
Evaluate piecewise functions	https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:absolute-value-piecewise-functions/x2f8bb11595b61c86:piecewise-functions/v/evaluating-piecewise-functions-example
Piecewise functions graphs	https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:absolute-value-piecewise-functions/x2f8bb11595b61c86:piecewise-functions/v/graphing-piecewise-function
Completing the Square	
Completing the square (intermediate)	https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:completing-square-quadratics/v/rewriting-quadratics-as-perfect-squares
Writing Equation of a Line: Point Slope Form	
Point-slope form	https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:forms-of-linear-equations/x2f8bb11595b61c86:point-slope-form/v/point-slope-and-slope-intercept-form-from-two-points
Rationalizing the Denominator	
Rationalising the denominator (advanced)	Click on hint button inside the practice

Mandatory Summer Assignment Part 1, Required Trig Exercises Answer Key

A. **Evaluate each trigonometric function for the value or angle given.**

$\sec(2\pi)$= **1 because** $\boldsymbol{sec(2\pi) = \frac{1}{cos\,(2\pi)} = \frac{1}{1} = 1}$

$\csc\pi$=undefined **because** $\boldsymbol{\csc(\pi) = \frac{1}{sin\,(\pi)} = \frac{1}{0} = undef.}$

$sin^{-1}(\frac{\sqrt{3}}{2})$= **π/3**

$cos^{-1}(\frac{-\sqrt{3}}{2})$= **5 π/6**

$\csc\frac{-7\pi}{6}$= **2 because** $\boldsymbol{\csc\left(\frac{-7\pi}{6}\right) = \frac{1}{sin\,(\frac{-7\pi}{6})} = \frac{1}{\frac{1}{2}} = 2}$

$\sec\frac{-5\pi}{6}$= $\boldsymbol{-2\sqrt{3}/3}$

$\tan\frac{5\pi}{3}$= **-** $\boldsymbol{\sqrt{3}}$

$\cot\frac{3\pi}{4}$= **-1**

$\arccos\frac{-1}{2}$= $\boldsymbol{\frac{2\pi}{3}}$ **because the restricted domain of arccos x is** $\boldsymbol{[0, \pi]}$

$\arcsin\frac{-1}{2}$= **- π/6 because the restricted domain of arcsin x is** $\boldsymbol{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$

$\arctan(\sqrt{3}) = {}^{\boldsymbol{\pi}}/_{\boldsymbol{3}}$ **because the restricted domain of arctan x is** $\boldsymbol{(-\frac{\pi}{2}, \frac{\pi}{2})}$ **and tan x is defined as opp/adj. Use a 30-60-90 triangle.**

$\arctan(\frac{\sqrt{3}}{3})$= $\boldsymbol{\pi/6}$ **because the restricted domain of arctan x is** $\boldsymbol{(-\frac{\pi}{2}, \frac{\pi}{2})}$ **and tan x is defined as opp/adj. Use a 30-60-90 triangle.**

B. Solve the equation $2\,sin^2 x + sinx = 1$ on the interval $[0,2\pi]$ **: x= π/6, x= 5π/6, x=3π/2**

$$2\,sin^2 x + sinx = 1$$
$$2\,sin^2 x + sinx - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$$2\sin x - 1 = 0 \quad OR \quad \sin x + 1 = 0$$
$$\sin x = \frac{1}{2} \qquad OR\ \sin x = -1 \qquad use\ your\ knowledge\ of\ Unit\ Circle\ on\ the\ given\ domain\ of\ [0,2\pi] \text{ to } solve\ for\ x\ such\ that\ each\ equation\ is\ true.$$

C. Solve the equation $2\,cos^2 x + cosx = 1$ on the interval $[0,2\pi]$: **x= π/3, x= 5π/3, x=π**

$2\,cos^2 x + \cos x = 1$
$2\,cos^2 x + \cos x - 1 = 0$
$(2\cos x - 1)(\cos x + 1) = 0$
$2\cos x - 1 = 0 \;\; OR \;\; \cos x + 1 = 0$

$\cos x = \frac{1}{2} \quad OR \;\; \cos x = -1 \quad use\ your\ knowledge\ of\ Unit\ Circle\ on\ the\ given\ domain\ of\ [0,2\pi]$ to $solve\ for\ x\ such\ that\ each\ equation\ is\ true$

D. Solve the equation $\sin 2x = -1\ on\ the\ interval\ [0,3\pi]$: **x= 3π/4, x= 7π/4, x=11π/4**

In order for sin x to be -1, the input angle must be $\frac{3\pi}{2}\ or\ \frac{7\pi}{2}\ or\ \frac{11\pi}{2}$ therefore set up the equations $2x = \frac{3\pi}{2}\ or\ 2x = \frac{7\pi}{2}\ or\ 2x = \frac{11\pi}{2}$ to solve for x *such that the equation is true on the given domain* $[0,3\pi]$. The function $\sin 2x$ has a period of π, so you will have a 3 complete sine curves on $[0,3\pi]$.

E. Solve the equation 2sin 2x = 1 on the interval $[0,2\pi]$: **x= π/12, x= 5π/12, x=13π/12, x=17π/12**

In order for sin x to be ½ , the input angle must be $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ therefore set up the equations $2x = \frac{\pi}{6},\ 2x = \frac{5\pi}{6}, 2x = \frac{13\pi}{6}, 2x = \frac{17\pi}{6}$ to solve for x *such that the equation is true on the given domain* $[0,2\pi]$. The function $\sin 2x$ has a period of π, so you will have a 2 complete sine curves on $[0,2\pi]$.

F. Solve the equation $sin^2 x - sinx = 0$ on the interval $[0,2\pi]$: **x= 0, x= π, x=2π, x=π/2**

$sin^2 x - sinx = 0$
$\sin x\,(\sin x - 1) = 0$
$sin\ x = 0$ OR $sin\ x = 1$ use your knowledge of Unit Circle on the given domain of $[0,2\pi]$ to solve for x such that each equation is true.

G. Solve the equation $cos^2 x - cosx = 0$ on the interval $[0,2\pi]$: **x= 0, x=2π, x= π/2, x=3π/2**

$cos^2 x - \cos x = 0$

$\cos x\,(\cos x - 1) = 0$

$cos\ x = 0$ OR $cos\ x = 1$ use your knowledge of Unit Circle on the given domain of $[0,2\pi]$ to solve for x such that each equation is true.

Next set of exercises: **<u>Optional summer assignment (key at the end)</u>**

Graphing: Sketch the graph and state the Domain & Range. Use interval notation for the Domain & Range

1. $y = x$ Domain: Range:	2. $y = x^2$ Domain: Range:	3. $x = y^2$ Domain: Range:
4. $y = \frac{1}{x}$ Domain: Range:	5. $y = \sqrt{x}$ Domain: Range:	6. $y = e^x$ Domain: Range:
7. $y = \ln x$ Domain: Range:	8. $y = \sin x$ Domain: Range:	9. $y = \cos x$ Domain: Range:
10. $y = \lvert x \rvert$ Domain: Range:	11. $x^2 = 16 - y^2$ Domain: Range:	12. $y = \sqrt{4 - x^2}$ Domain: Range:

1. Evaluating Functions:

A)

$$f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$$

(a) $f(-2)$ (b) $f(1)$ (c) $f(2)$

B)

$$f(x) = \begin{cases} 3x - 1, & x < -1 \\ 4, & -1 \le x \le 1 \\ x^2, & x > 1 \end{cases}$$

(a) $f(-2)$ (b) $f\left(-\frac{1}{2}\right)$ (c) $f(3)$

C)

$$f(s) = \frac{|s - 2|}{s - 2}$$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$					

II. Algebraic Manipulation

A. Factor each of the following completely or state that it is prime.

13. $x^2 + 5x + 6$

14. $x^2 - 4x - 12$

15. $x^2 + 5x - 24$

16. $16x^2 - 81$

17. $x^3 + 4x^2 + 3x$

18. $x^4 + 6x^2 + 9$

A'. Completing the square: Find h and k such that $x^2 + 12x + 4 = (x + h)^2 + k$

B. Simplify

19. $\dfrac{(x+4)^2-16x}{x-4}$

20. $\dfrac{y+3}{(y+4)^2-(8y+25)}$

21. $\dfrac{\frac{25}{a}-a}{5+a}$

22. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

C. Solve for x *without* a calculator or grapher:

23. $1+\frac{x}{3}=\frac{4}{5}$	24. $\frac{2}{3}-\frac{5}{7}=x$	25. $\frac{3}{8}=\frac{4}{1-x}$
26. $\frac{6}{x}+\frac{x}{2}=4$	27. ln (x-2) = 4	28. -3ln(x+1) = 2
29. ln(x-2) + ln(3)= 5	30. $4e^{3x-2}=8$	31. $100=200e^{-0.06x}$

32. $\sin x = -\frac{\sqrt{2}}{2}$, $for\, 0 \le x \le 2\pi$	33. $3x^2 + 7x + 3 = 0$

D. Find the asymptotes (horizontal, vertical). Show work.

34. $y = \frac{1}{x-1}$

35. $y = \frac{1}{(x-1)^2}$

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A) $y = \frac{x-1}{x^3-1}$

B) $y = \frac{x^2-3x+2}{x^2+2x-3}$

E. Equations of Lines

36. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

37. Using point slope form, find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

38. Using point slope form, find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

F. Function Notation

Let $f(x)=x^2,\ g(x)=2x+5,\ \ and\ \ h(x)=x^2-1$. **Find each.**

39. $h\left[f(-2)\right]=$ ______

40. $f\left[g(x-1)\right]=$ ______

41. $g\left[h(x^3)\right]=$ ______

G. Modeling Equations

42. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

43. The sum of the perimeters of an equilateral triangle and a square is 10. Let s be the side length of the equilateral triangle and let x be the side length of the square. Find the equation for the area of the square in terms of s.

44. A manufacturer makes a metal can in the shape of a cylinder that holds $1000\,\text{cm}^3$ of oil. Write an equation for the Surface area of the can in terms of one variable.

H. Difference Quotient:

$$f(x) = x^2 - x + 1, \qquad \frac{f(2+h) - f(2)}{h}, h \neq 0$$

$$f(x) = 5x - x^2, \qquad \frac{f(5+h) - f(5)}{h}, h \neq 0$$

I. Division of Polynomials:

A) $(x^3 - 2x^2 - 5x + 6) \div (x - 3)$

B) $(2x^4 + 3x^3 + 5x - 1) \div (x^2 + 3x + 2)$

J. Exponents and Logarithmic properties:

a) $\boldsymbol{Simplify}$: $\left(\frac{2x^4 3y^6}{4xy^4}\right)^{-2}$

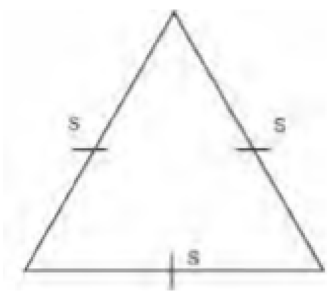
b) $\boldsymbol{Expand}$: $\boldsymbol{ln}\left(\frac{3x^2}{2y^3}\right)$

c) rewrite in exponential form: $\sqrt[3]{2x^5}$

d) Evaluate: e^{3lne^2}

III. Area/Geometry Questions

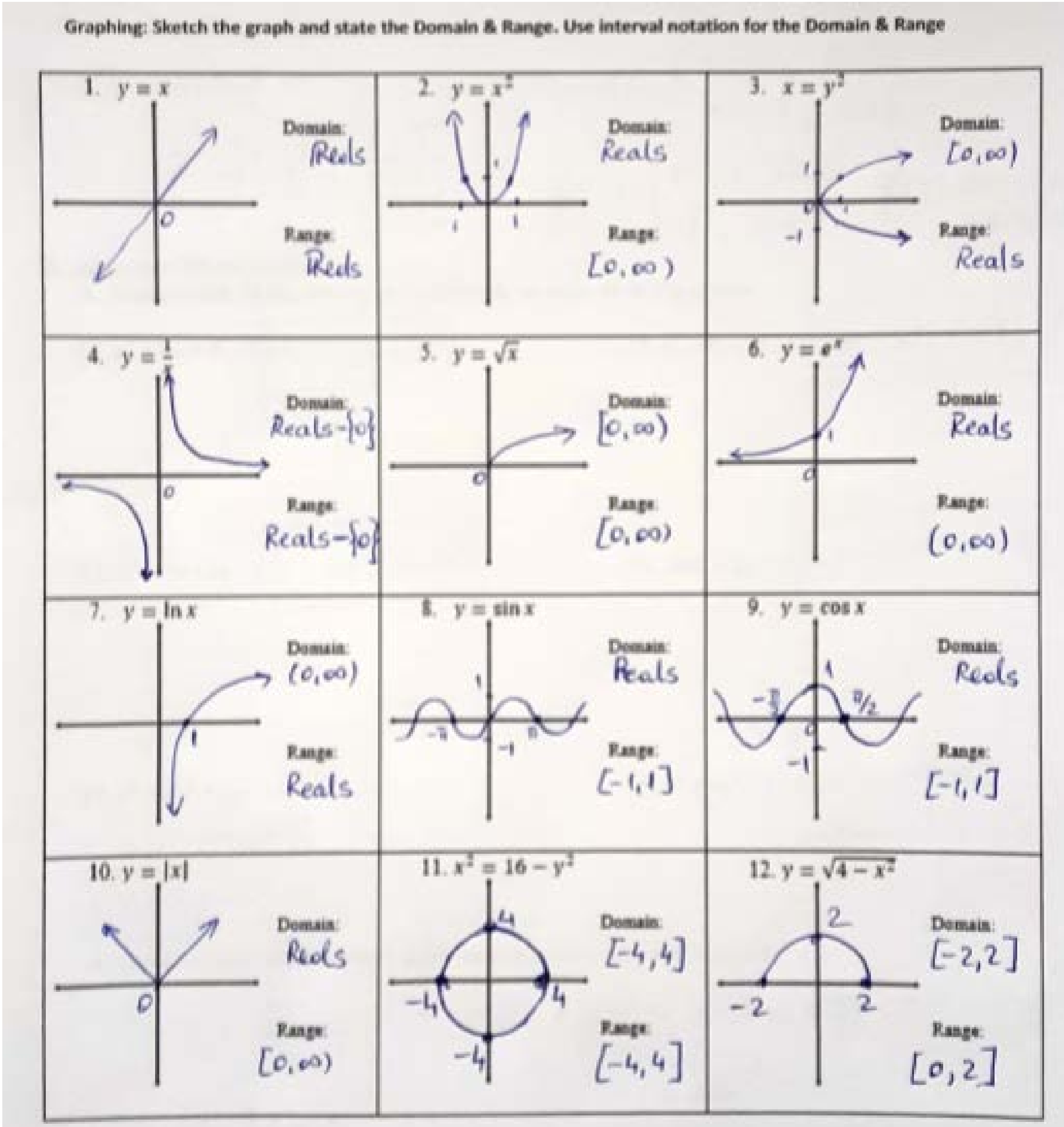
45. What is the area of an equilateral triangle with side length s? Hint: use a 30-60-90 triangle!



46. What is the area of a semicircle with respect to diameter d?

47. What's the area of an isosceles right triangle with base $(4-x^2)$?

KEY for the: **<u>Optional summer assignment</u>**



Graphing: Sketch the graph and state the Domain & Range. Use interval notation for the Domain & Range

1. $y = x$ Domain: Reals Range: Reals	2. $y = x^2$ Domain: Reals Range: $[0, \infty)$	3. $x = y^2$ Domain: $[0, \infty)$ Range: Reals
4. $y = \frac{1}{x}$ Domain: Reals$-\{0\}$ Range: Reals$-\{0\}$	5. $y = \sqrt{x}$ Domain: $[0, \infty)$ Range: $[0, \infty)$	6. $y = e^x$ Domain: Reals Range: $(0, \infty)$
7. $y = \ln x$ Domain: $(0, \infty)$ Range: Reals	8. $y = \sin x$ Domain: Reals Range: $[-1, 1]$	9. $y = \cos x$ Domain: Reals Range: $[-1, 1]$
10. $y = \lvert x \rvert$ Domain: Reals Range: $[0, \infty)$	11. $x^2 = 16 - y^2$ Domain: $[-4, 4]$ Range: $[-4, 4]$	12. $y = \sqrt{4 - x^2}$ Domain: $[-2, 2]$ Range: $[0, 2]$

1. **Evaluating Functions:**

A) $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2)$ (b) $f(1)$ (c) $f(2)$

a) $f(-2) = (-2)^2 + 2 = \boxed{6}$

b) $f(1) = 1^2 + 2 = \boxed{3}$

c) $f(2) = 2 \cdot 2^2 + 2 = \boxed{10}$

B) $f(x) = \begin{cases} 3x - 1, & x < -1 \\ 4, & -1 \le x \le 1 \\ x^2, & x > 1 \end{cases}$

(a) $f(-2)$ (b) $f\left(-\frac{1}{2}\right)$ (c) $f(3)$

a) $f(-2) = 3(-2) - 1 = \boxed{-7}$

b) $f\left(-\frac{1}{2}\right) = \boxed{4}$

c) $f(3) = 3^2 = \boxed{9}$

C) $f(x) = \frac{|x - 2|}{x - 2}$

x	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(x)$	-1	-1	-1	1	1

$|s - 2| = \begin{cases} s - 2, & s \ge 2 \\ -s + 2, & s < 2 \end{cases}$

$f(s) = \begin{cases} 1, & s > 2 \\ -1, & s < 2 \end{cases}$ $f(2)$ does not exist (DNE)

II. Algebraic Manipulation

A. Factor each of the following completely or state that it is prime.

13. x^2+5x+6 $= \boxed{(x+2)(x+3)}$

14. $x^2-4x-12$ $= \boxed{(x-6)(x+2)}$

15. $x^2+5x-24$ $= \boxed{(x+8)(x-3)}$

16. $16x^2-81$ $= \boxed{(4x-9)(4x+9)}$

17. x^3+4x^2+3x

$= x(x^2+4x+3)$

$= \boxed{x(x+3)(x+1)}$

$x^2 = u$

18. $x^4+6x^2+9 = u^2+6u+9$

$=(u+3)^2 = \boxed{(x^2+3)^2}$

A'. Completing the square: Find h and k such that $x^2+12x+4=(x+h)^2+k$

$h = \frac{12}{2} = 6$

$6^2 = 36$

$x^2+12x+4 = (x^2+12x+\underline{36})+4-\underline{36}$

$=(x+6)^2-32$

$\boxed{h=6 \quad , \quad k=-32}$

B. Simplify

19. $\dfrac{(x+4)^2-16x}{x-4} = \dfrac{x^2+8x+16-16x}{x-4}$

$= \dfrac{x^2-8x+16}{x-4} = \dfrac{(x-4)^2}{x-4} = \boxed{x-4}$

20. $\dfrac{y+3}{(y+4)^2-(8y+25)} = \dfrac{y+3}{y^2+8y+16-8y-25} =$

$= \dfrac{y+3}{y^2-9} = \dfrac{y+3}{(y+3)(y-3)} = \boxed{\dfrac{1}{y-3}}$

21. $\dfrac{\frac{25}{a}-a}{5+a} = \dfrac{25-a^2}{a(5+a)} = \dfrac{(5+a)(5-a)}{a(5+a)}$

$= \boxed{\dfrac{5-a}{a}}$

22. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}} = \dfrac{x^2-x-1}{x(x+1)} \div \dfrac{x^2+x+1}{x(x+1)}$

$= \boxed{\dfrac{x^2-x-1}{x^2+x+1}}$

C. Solve for x *without* a calculator or grapher:

23. $1+\frac{x}{3}=\frac{4}{5}$ $15+5x=12$ $5x=-3$ $\boxed{x=-\frac{3}{5}}$	24. $\frac{2}{3}-\frac{5}{7}=x$ $14-15=21x$ $\boxed{\frac{-1}{21}=x}$	25. $\frac{3}{8}=\frac{4}{1-x}$ $3-3x=32$ $-3x=29$ $\boxed{x=-\frac{29}{3}}$
26. $\frac{6}{x}+\frac{x}{2}=4$ $\quad x\neq 0$ $12+x^2=8x$ $x^2-8x+12=0$ $(x-2)(x-6)=0$ $\boxed{x=2}$, $\boxed{x=6}$	27. $\ln(x-2)=4$ $x-2=e^4$ $\boxed{x=2+e^4}$	28. $-3\ln(x+1)=2$ $\ln(x+1)=-\frac{2}{3}$ $x+1=e^{-2/3}$ $\boxed{x=-1+e^{-2/3}}$
29. $\ln(x-2)+\ln(3)=5$ $\ln(3x-6)=5$ $3x-6=e^5$ $3x=6+e^5$ $\boxed{x=(6+e^5)/3}$	30. $4e^{3x-1}=8$ $e^{3x-1}=2$ $3x-1=\ln 2$ $3x=1+\ln 2$ $\boxed{x=(1+\ln 2)/3}$	31. $100=200e^{-0.06x}$ $\frac{1}{2}=e^{-0.06x}$ $\ln(\frac{1}{2})=-0.06x$ $-\ln 2=-0.06x$ $\boxed{x=\ln 2/0.06}$

32. $\sin x = -\frac{\sqrt{2}}{2}$, for $0 \le x \le 2\pi$

$\sin x < 0$ in 3^{rd} & 4^{th} quadrants

reference angle : $\frac{\pi}{4}$

$x = \pi + \frac{\pi}{4} = \boxed{\frac{5\pi}{4}}$; $x = 2\pi - \frac{\pi}{4} = \boxed{\frac{7\pi}{4}}$

33. $3x^2 + 7x + 3 = 0$

$$x = \frac{-7 \pm \sqrt{49 - 4 \cdot 3 \cdot 3}}{2 \cdot 3}$$

$$\boxed{x = \frac{-7 \pm \sqrt{13}}{6}}$$

D. Find the asymptotes (horizontal, vertical). Show work.

34. $y = \frac{1}{x-1}$

$x=1$ vertical asymptote

$y=0$ horizontal asymptote

35. $y = \frac{1}{(x-1)^2}$

$x=1$ V.A.

$y=0$ H.A.

D'. Find the holes (if any). Show work.

A) $y = \frac{x-1}{x^3-1}$

$f(x) = y = \frac{\cancel{x-1}}{\cancel{(x-1)}(x^2+x+1)} \rightarrow \frac{1}{x^2+x+1} = g(x)$

$g(1) = \frac{1}{3}$; $f(x) = g(x)$ except for $x=1$

$f(1) = \text{DNE}$

$\boxed{f \text{ has a hole @ } \left(1, \frac{1}{3}\right)}$

B) $y = \frac{x^2-3x+2}{x^2+2x-3}$

$f(x) = y = \frac{\cancel{(x-1)}(x-2)}{\cancel{(x-1)}(x+3)} \rightarrow$

$g(x) = \frac{x-2}{x+3}$ $g(1) = \frac{-1}{4}$

$f(1) = \text{DNE}$, $f(x) = g(x)$ except for $x=1$

$\boxed{f \text{ has a hole @ } \left(1, -\frac{1}{4}\right)}$

E. Equations of Lines

36. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

$\boxed{y-5=\frac{2}{3}(x-0)}$ OR $y=\frac{2}{3}x+5$

37. Using point slope form, find the equation of a line passing through the point (2, 8) and parallel to the line $y=\frac{5}{6}x-1$.

$\boxed{y-8=\frac{5}{6}(x-2)}$ or $y=8+\frac{5}{6}(x-2)$

38. Using point slope form, find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

$\boxed{y=7}$

↳ horizontal line

F. Function Notation

Let $f(x)=x^2$, $g(x)=2x+5$, *and* $h(x)=x^2-1$. **Find each.**

39. $h[f(-2)]=\underline{h(4)}=16-1=\boxed{15}$

40. $f[g(x-1)]=\underline{f(2}(x-1)+5)=f(2x+3)=\boxed{(2x+3)^2}$

41. $g[h(x^3)]=\underline{g(}(x^3)^2-1)=g(x^6-1)=2(x^6-1)+5=\boxed{2x^6+3}$

G. Modeling Equations

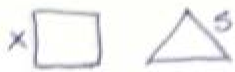
42. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

$$\begin{cases} x \cdot y = 192 \\ x + 3y = \text{min.} \end{cases}$$

$\cdot$ 192		$x+3y$ (min)
$2 \cdot 86$	$\longrightarrow$	104
$3 \cdot 64$	$\longrightarrow$	73
$4 \cdot 48$	$\longrightarrow$	50
$6 \cdot 32$	$\longrightarrow$	50
$8 \cdot 24$	$\longrightarrow$	48
$12 \cdot 16$	$\longrightarrow$	52

$x+3y$ min. when $\boxed{y=8,\ x=24}$

43. The sum of the perimeters of an equilateral triangle and a square is 10. Let s be the side length of the equilateral triangle and let x be the side length of the square. Find the equation for the area of the square in terms of s.



$$4x + 3s = 10 \longrightarrow x = (10-3s)/4$$

$$\text{Area (square)} = x^2 = \left(\frac{10-3s}{4}\right)^2$$

44. A manufacturer makes a metal can in the shape of a cylinder that holds $1000\,cm^3$ of oil. Write an equation for the Surface area of the can in terms of one variable.

$$V_{cyl.} = \pi r^2 h = 1000\ cm^3 \rightarrow h = \frac{1000}{\pi r^2}$$

$$\text{Surface Area}_{cyl.} = 2\pi r h = 2\pi r \cdot \frac{1000}{\pi r^2} = \boxed{\frac{2000}{r}\ cm^2}$$

H. Difference Quotient:

a) $f(x) = x^2 - x + 1, \quad \frac{f(2+h) - f(2)}{h}, h \neq 0$

b) $f(x) = 5x - x^2, \quad \frac{f(5+h) - f(5)}{h}, h \neq 0$

a) $\frac{(2+h)^2-(2+h)+1-(2^2-2+1)}{h} = \frac{\cancel{2^2}+4h+h^2-\cancel{2}-h+\cancel{1}-\cancel{2^2}+\cancel{2}-\cancel{1}}{h} = \frac{h^2+3h}{h} = \frac{h(h+3)}{h} = \boxed{h+3}$

b) $\frac{5(5+h)-(5+h)^2-(5.5-5^2)}{h} = \frac{\cancel{5.5}+5h-\cancel{5^2}-10h-h^2-\cancel{5.5}+\cancel{5^2}}{h} = \frac{-h^2-5h}{h} = \boxed{-h-5}$

I. Division of Polynomials:

A) $(x^3 - 2x^2 - 5x + 6) \div (x - 3)$

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -5 & 6 \\ & & 3 & 3 & -6 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$\rightarrow \boxed{x^2 + x - 2}$

B) $(2x^4 + 3x^3 + 5x - 1) \div (x^2 + 3x + 2)$

$2x^2 - 3x + 5$ = Quotient

$$\begin{array}{rl} x^2+3x+2 \,\big) & 2x^4 + 3x^3 + 5x - 1 \\ - & 2x^4 + 6x^3 + 4x^2 \\ \hline & -3x^3 - 4x^2 + 5x - 1 \\ - & -3x^3 - 9x^2 - 6x \\ \hline & 5x^2 + 11x - 1 \\ - & 5x^2 + 15x + 10 \end{array}$$

$\rightarrow$ Remeinder $= \boxed{-4x - 11}$

J. Exponents and Logarithmic properties:

a) ***Simplify***: $\left(\frac{2x^4 3y^6}{4xy^4}\right)^{-2} = \boxed{\frac{4}{9x^6y^4}}$

b) ***Expand***: $ln\left(\frac{3x^2}{2y^3}\right)$

$= \boxed{\ln 3 + 2\ln x - \ln 2 - 3\ln y}$

c) rewrite in exponential form: $\sqrt[3]{2x^5}$

$$= \boxed{2^{1/3} \cdot x^{5/3}}$$

d) Evaluate: $e^{3\ln e^2}$ $= e^{\ln(e^2)^3} = e^{\ln e^6} = \boxed{e^6}$

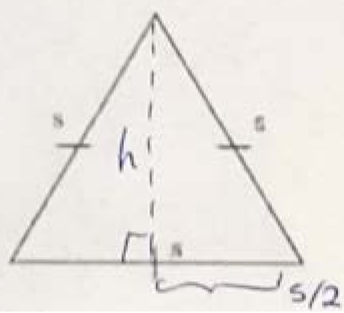
OR

$$e^{3\cdot 2\ln e} = e^{6\ln e} = e^{6}$$

$$(\ln e = 1)$$

III. Area/Geometry Questions

45. What is the area of an equilateral triangle with side length s? Hint: use a 30-60-90 triangle!



$$\text{Area} = \frac{1}{2} \cdot s \cdot h$$

$$h^2 + \left(\frac{s}{2}\right)^2 = s^2 \rightarrow h^2 = s^2 - \frac{s^2}{4} = \frac{3s^2}{4} \rightarrow h = \frac{s\sqrt{3}}{2}$$

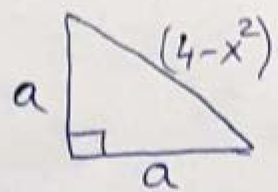
$$\text{Area} = \frac{1}{2} \cdot s \cdot \frac{s\sqrt{3}}{2} = \frac{s^2\sqrt{3}}{4}$$

46. What is the area of a semicircle with respect to diameter d?

$$\text{Area (semicircle)} = \frac{\pi r^2}{2} = \frac{\pi}{2}\left(\frac{d}{2}\right)^2 = \frac{\pi}{2} \cdot \frac{d^2}{4} = \frac{\pi d^2}{8}$$

$$r = \frac{d}{2}$$

47. What's the area of an isosceles right triangle with base $(4\text{-}x^2)$?



$$2a^2 = (4-x^2)^2 \rightarrow a^2 = \frac{(4-x^2)^2}{2}$$

$$\text{Area}_\triangle = \frac{1}{2} \cdot a^2 = \frac{1}{2} \cdot \frac{(4-x^2)^2}{2}$$

$$\boxed{\text{Area} = \frac{(4-x^2)^2}{4}}$$