

Problem solving challenge #3

1. Solve the equation $x^2 + \frac{x^2}{(x+1)^2} = 3$

2.

Luke noticed that if you take any three consecutive integers, multiply them together, and add the middle number to the result, the answer always seems to be the middle number cubed.

For example:

$$3 \times 4 \times 5 + 4 = 64 = 4^3$$
$$4 \times 5 \times 6 + 5 = 125 = 5^3$$
$$9 \times 10 \times 11 + 10 = 1000 = 10^3$$

- a. In order prove his observation true, Luke writes down $(n + 1)(n + 2)(n + 3) + (n + 2)$. What answer is he hoping to show this expression equals?
- b. Lulu, upon hearing of Luke's observation, writes down her own version with n as the middle number. What does her formula look like?
- c. Use Lulu's expression to prove that adding the middle number to the product of any three consecutive numbers is sure to equal that middle number cubed.