

Problem solving challenge #2

1. Determine all real solutions to the system of equations:

$$x^2 + y^2 + x + y = 12$$

$$xy + x + y = 3$$

(Hint: there will be three solutions)

2. Carmen selects four different numbers from the set $\{1, 2, 3, 4, 5, 6, 7\}$ whose sum is 11. If l is the largest of these four numbers, what is the value of l ?

3. Let x, y and z be positive real numbers. Show that $x^2 + xy^2 + xyz^2 \geq 4xyz - 4$.

(Hint: use $x^2 + 4 > 4x$; equal for $x=2$; $4x + xy^2 > 4xy$; $4xy + xyz^2 > 4xyz$, then add all inequalities)

4. If $x = \left(1 + \frac{1}{n}\right)^n$ and $y = \left(1 + \frac{1}{n}\right)^{n+1}$, show that $y^x = x^y$.

(Hint: apply log to relationship and simplify)