Problem solving

1. Each of six fruit baskets contains pears, plums and apples. The number of plums in each basket equals the total number of apples in all other baskets combined while the number of apples in each basket equals the total number of pears in all other baskets combined. Prove that the total number of fruits is a multiple of 31.

2.

Factor:

$$4x^5 + 4x^4 - 149x^3 - 149x^2 + 1225x + 1225$$

Your answer should have only linear factors!!!

3.

Let $\dfrac{x^2+y^2}{x^2-y^2} + \dfrac{x^2-y^2}{x^2+y^2} = k$.

Compute the following expression in terms of $k$:

$$E(x,y) = \frac{x^8+y^8}{x^8-y^8} - \frac{x^8-y^8}{x^8+y^8}.$$

4. Let $x$ and $y$ be positive reals such that $x^3 + y^3 + (x+y)^3 + 30xy = 2000.$

Show that $x + y = 10$. ( hint: use sum of cubes and factoring)